

A Simple Adaptive First-Order Differential Microphone

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1 Abstract

This talk covers the design and implementation of a novel adaptive first-order differential microphone that minimizes the microphone output power under the constraint that the solitary first-order microphone null is located in the rear-half plane. Although the solution presented does not maximize the signal-to-noise ratio, it can significantly improve the signal-to-noise ratio in certain acoustic fields. An adaptive differential microphone has been implemented by combining two omnidirectional elements to form back-to-back cardioid directional microphones. By combining the weighted subtraction of these two outputs, any first-order array can be realized. If certain constraints are placed on the combination weighting, the null location can be constrained to defined angular regions.

2 Introduction

As communication devices become more portable and used in any environment, the acoustic pick-up by electroacoustic transducers will require the combination of small compact transducers and signal-processing to allow high quality communication. One of the basic problems in acoustic transduction for personal communication devices is the detrimental effect of background noise. Also, reverberation can seriously degrade the microphone reception of speech signals in hands-free applications. Directional microphone arrays can be effective in combating both of these problems. One particularly attractive microphone design for personal communicators and teleconferencing is the differential microphone array. This type of array uses sensors that are spaced very closely compared to the acoustic wavelength. To realize directionality, the elements are combined in an alternating sign fashion and as a result of the close-spacing, can be seen to be a differential array. The resulting differential sensor is superdirectional since the directivity is higher than that of the uniformly summed output of all the sensor elements. Typically, optimal directional microphone are developed designed under the assumption that the acoustic reverberation or noise fields are isotropic. Unfortunately, real acoustic noise fields never match ideal theoretical assumptions. A better potential solution is to design an adaptive microphone system that adjusts its directivity pattern to maximize the signal-to-noise ratio. Much work has been done on adaptive beamforming in the past three decades [1, 2, 3, 4], however, little attention has been given to adaptive differential arrays. One reason for the lack of attention towards differential sensors is the well known fact that superdirectional arrays are extremely difficult to realize practically. However, if the differential order of the sensor is limited to first or second-order, then practical designs do exist.

3 Derivation of the adaptive first-order array

For a plane-wave signal $s(t)$ with spectrum $S(\omega)$ and wavevector \mathbf{k} incident on a two-element array with displacement vector \mathbf{d} as shown in Fig. 1, the output can be written as,

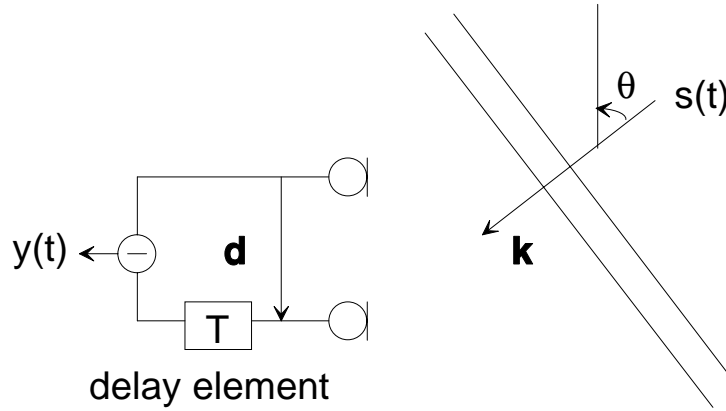


Figure 1: Diagram of first-order sensor composed of two zero-order sensors and a delay.

$$\begin{aligned} Y(\omega, \theta) &= S(\omega) \left(1 - e^{-j(\omega T + \mathbf{k} \cdot \mathbf{d})} \right) \\ &= S(\omega) \left(1 - e^{-j\omega [T + (d \cos \theta)/c]} \right) \end{aligned} \quad (1)$$

where d is the interelement spacing, T is equal to the delay applied to the signal from one sensor and we have made the substitution $|\mathbf{k}| = k = \omega/c$ where c is the speed of sound. Taking the magnitude of Eq. 1 yields,

$$|Y(\omega, \theta)| = 2 \left| S(\omega) \sin \frac{\omega [T + (d \cos \theta)/c]}{2} \right|. \quad (2)$$

If we assume a small spacing and delay ($kd \ll \pi$ and $\omega T \ll \pi$),

$$|Y(\omega, \theta)| \approx \omega |S(\omega) [T + (d \cos \theta)/c]|. \quad (3)$$

As expected, the first-order differential array has a monopole term and a first-order dipole term $\cos \theta$ which resolves the component of the acoustic particle velocity along the sensor axis. One thing to notice from Eq. 3 is that the first-order array has a first-order differentiator frequency dependence (increases

linearly with frequency). This frequency dependence is easily compensated in practice by a first-order lowpass filter. The term in the brackets in Eq. 3 contains the array directional response.

One particularly elegant way to implement a general first-order differential microphone is the scalar combination of two back-to-back cardioid microphones. The back-to-back cardioid arrangement can easily be implemented as shown in Fig. 2. The lowpass filter shown following the output $y(t)$ in Fig. 2 is used to compensate the differentiator response of the differential microphone. By setting the sampling period equal to d/c we can form the back-to-back cardioid microphone outputs directly. Fig. 3 is a plot which

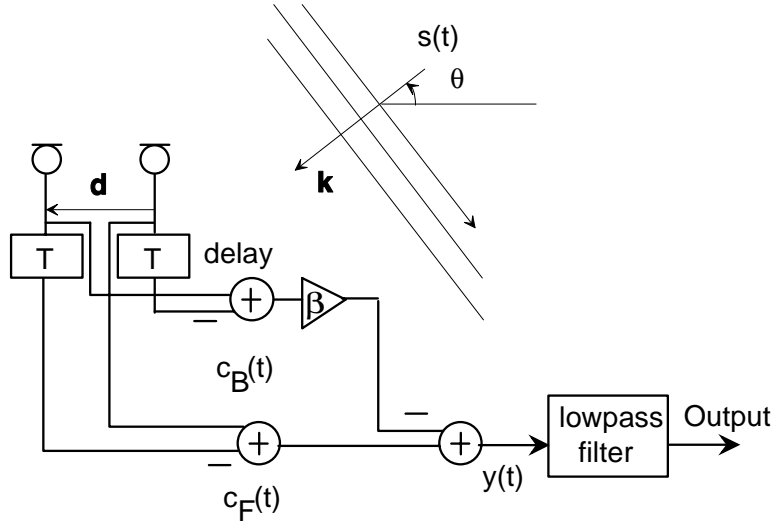


Figure 2: Schematic implementation of an adaptive first-order differential microphone using the combination of a forward and backward facing cardioids.

shows the directivity patterns of the back-to-back cardioid arrangement.

By examining Fig. 2 and Eq. 1 and noting that the sampling period $T = d/c$, we can write the expressions for the the forward facing cardioid C_F , and the backwards facing cardioid C_B , (note that to simplify the following expressions the spatial origin is placed at the array center):

$$C_F(\omega, \theta) = 2jS(\omega)e^{-j\omega T/2} \sin \frac{kd(1 + \cos \theta)}{2} \quad (4)$$

and,

$$C_B(\omega, \theta) = 2jS(\omega)e^{-j\omega T/2} \sin \frac{kd(1 - \cos \theta)}{2}. \quad (5)$$

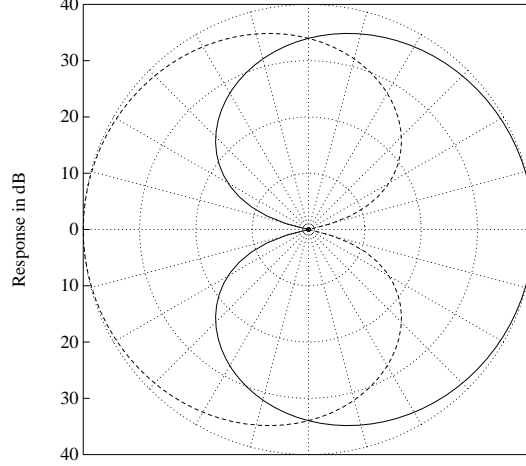


Figure 3: Directional responses of the back-to-back cardioid transducer as shown in Fig. 2

Normalizing the output signal by the input spectrum $S(\omega)$ results in,

$$\left| \frac{Y(\omega, \theta)}{S(\omega)} \right| = 2 \left| \sin \frac{kd(1 + \cos \theta)}{2} - \beta \sin \frac{kd(1 - \cos \theta)}{2} \right| \quad (6)$$

4 Optimum β

The optimum value of β is defined as the value of β which minimizes the mean-square value of the sensor output. Starting with the time-domain representation of the back-to-back cardioid as seen in Fig. 2, we can write,

$$y(t) = c_F(t) - \beta c_B(t). \quad (7)$$

Squaring and taking the expected value yields,

$$E[y^2(t)] = R_{c_F c_F}(0) - 2\beta R_{c_F c_B}(0) + \beta^2 R_{c_B c_B}(0) \quad (8)$$

where, $R_{c_F c_F}(0)$ and $R_{c_B c_B}(0)$ are the powers of the front and back cardioid signals and, $R_{c_F c_B}(0)$ is the cross-power between the front and back cardioid signals. Since the output power $E[y^2(t)]$ is a quadratic function of the variable β , we can be certain that there is only one minimum. The minimum value

can be found by taking the derivative of Eq. 8 with respect to β and setting the result to zero. The result is,

$$\beta_{opt} = \frac{R_{c_F c_B}(0)}{R_{c_B c_B}(0)}. \quad (9)$$

The value of β_{opt} is a minimum since the second derivative with respect to β is positive. The second derivative is positive as it has the same sign as an autocorrelation function at zero lag. The autocorrelation function must be positive (or zero) for zero-lag and therefore the error surface $E[y^2(t)]$ is concave with one minimum. The result shown in Eq. 9 is the optimum Wiener filter for a filter of length one.

In an actual DSP implementation we must use estimates of the power and cross-power. This restriction is not as problematic as one may think. In fact, it is desirable to use short-time estimates of the correlation functions since we know a priori that the acoustic fields in which we intend to operate the adaptive microphone are nonstationary.

5 LMS version

The LMS or Stochastic Gradient algorithm is a commonly used adaptive algorithm due to its simplicity and ease of implementation [1]. We therefore develop the LMS algorithm for the back-to-back cardioid adaptive first-order differential array. To begin, recall that

$$y(t) = c_F(t) - \beta c_B(t). \quad (10)$$

Squaring Eq. 10 results in,

$$y^2(t) = c_F^2(t) - 2\beta c_F(t)c_B(t) + \beta^2 c_B^2(t). \quad (11)$$

The steepest descent algorithm finds a minimum of the error surface $E[y^2(t)]$ by stepping in the direction opposite to the gradient of the surface with respect to the weight parameter β . The steepest descent update equation can be written as,

$$\beta_{t+1} = \beta_t - \mu \frac{dE[y^2(t)]}{d\beta} \quad (12)$$

where, μ is the update step-size and the differential gives the gradient of the error surface $E[y^2(t)]$ with respect to β . The quantity that we want to minimize is the mean of $y^2(t)$ but the LMS algorithm uses the instantaneous estimate of the gradient, i.e., the expectation operation in Eq. 12 is not applied and the instantaneous estimate is used. Performing the differentiation

yields,

$$\begin{aligned}\frac{dy^2(t)}{d\beta} &= -2c_F(t)c_B(t) + 2\beta c_B^2(t) \\ &= -2y(t)c_B(t).\end{aligned}\tag{13}$$

Thus, we can write the LMS update equation as,

$$\beta_{t+1} = \beta_t + 2\mu y(t)c_B(t).\tag{14}$$

Typically the LMS algorithm is slightly modified by normalizing the update size so that explicit convergence bounds for μ can be stated that are independent of the input power. The LMS version with a normalized μ is therefore,

$$\beta_{t+1} = \beta_t + 2\mu y(t) \frac{c_B(t)}{\langle c_B^2(t) \rangle}\tag{15}$$

where the brackets indicate a time average.

6 Implementation and Measurements

An experimental adaptive array was realized using two Bruel&Kjær 4183 microphones spaced at 1.5 cm. The sampling rate was set to 22 kHz which allows the easy formation of the back-to-back cardioids by appropriately subtracting the present samples from the previous samples. A 5 kHz lowpass filter was utilized to eliminate signals that are above the highest design frequency for the array. A block diagram of the DSP program which implements the correlation-based estimate of β is shown in Fig. 4. One design aspect that should be noted in Fig. 4 is the use of saturation arithmetic for the weight β . By constraining the value of β to $0 \leq \beta \leq 1$, we are guaranteed that the null is in the rear-half plane. Of course we are assuming that the sensors are ideal and the source to be nulled is in the farfield. Since the adaptive array presented in the memorandum is designed to track in nonstationary acoustic sound fields, it is difficult to document the dynamic performance. We can, however, measure the directional responses of the array for various stationary noise fields as well as frequency responses. Fig. 5 shows directivity plots measured in an anechoic chamber. The plots are shown for a 2 kHz sinusoidal signal and for values of β which resulted in nulls being placed in approximately 10° increments.

7 3D steerable array

It is possible to expand the generality of the simple two-element adaptive microphone by utilizing more microphone elements. For general 3D steering

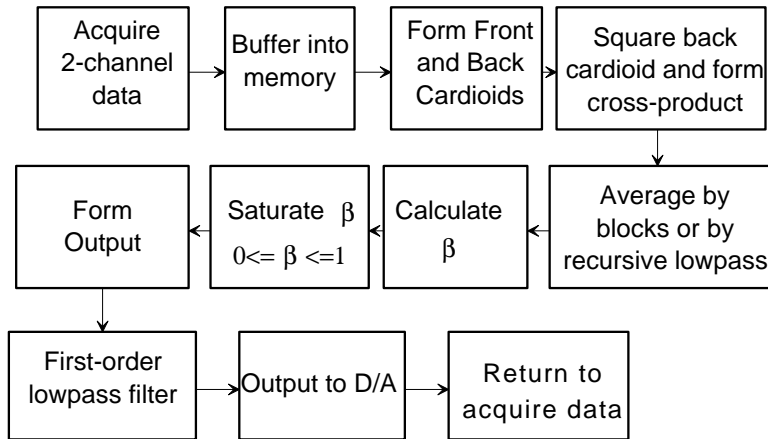


Figure 4: Block diagram of DSP program to implement the correlation-based adaptive differential microphone array

a minimum of four elements is required. One straightforward implementation is to use six elements whose positions are at the centers of the sides of a cube. A photograph of such a microphone array mounted in the rigid nylon sphere is shown in Fig. 6. A functional block diagram of the steerable first-order differential microphone is shown in Fig. 7. A general 3D steerable first-order differential adaptive microphone is obtained by combining the steered-dipole and the omnidirectional microphone signals to form the desired back-to-back cardioid signals. The back-to-back cardioid signals are then used as shown previously to form the steerable adaptive array output.

8 Conclusions

A simple first-order adaptive differential microphone with a null angle constrained to the rear-half plane has been presented. An array was constructed and the performance was shown to closely match the theoretical predictions. Two adaptation algorithms were presented: the Wiener optimal solution, and the LMS algorithm. A DSP implementation using an Lucent DSP32C was realized and a block diagram of the Wiener filter implementation given.

It is straightforward to implement a second-order array by using fixed first-order sensors instead of omnidirectional sensors. We have constructed such an array and qualitatively it appears to operate correctly. The adaptive

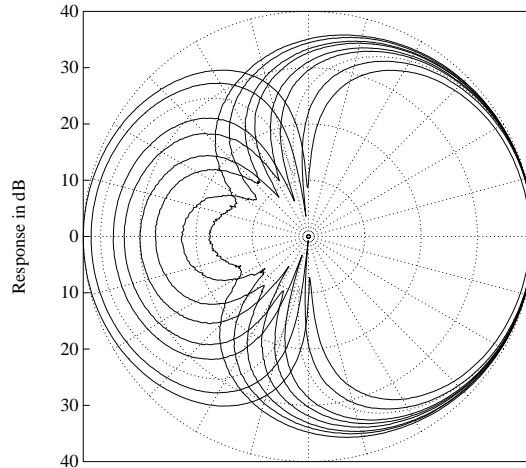


Figure 5: Measured directional responses for the adaptive array for $0 \leq \beta \leq 1$ and β chosen to give nulls in approximately 10° increments.

technique presented here is easily extrapolated to a second-order array with two independent nulls.

References

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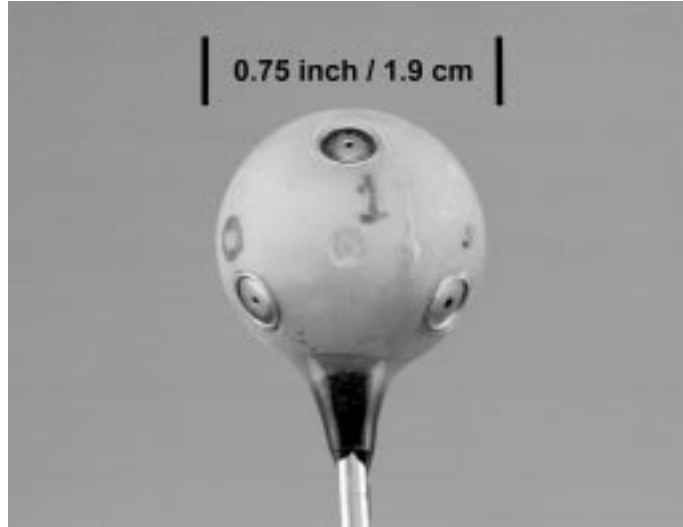


Figure 6: Photograph of the 6-element microphone array mounted in a 0.75 inch nylon sphere. Note that only 3 microphone capsules can be seen in this photograph (microphones 0,1, and 5).

