Presentation of the paper "The Generalized Correlation Method for Estimation of Time Delay"

Guided by:

Rajesh Hegde

Presented by:

Bhargava M S

Y8104013

Paper presents……

- Five Generalized Cross-Correlation function which differs in Weighting functions.
- Five processors are compared to outline their inherent relationships.
- Pros and cons of each processors.
- Interpretation of Low SNR of ML Estimator.

Need for Estimation of Time Delay

- First stage that feeds into subsequent processing blocks for identifying, localizing, and tracking radiating sources.
- Time delay estimation has been a research topic of significant practical importance in many fields (radar, sonar, seismology, geophysics, ultrasonics, hands-free communications, etc.)
- Target localization by sonar systems and position estimation by radio navigation systems.

Introduction

Two spatially seperated sensors can be mathematically modeled as below.

$$x_1(t) = s_1(t) + n_1(t)$$

$$x_2(t) = \alpha s_1(t + D) + n_2(t),$$

where $s_1(t)$, $n_1(t)$, and $n_2(t)$ are real, jointly stationary random processes. Signal $s_1(t)$ is assumed to be uncorrelated with noise $n_1(t)$ and $n_2(t)$.

$$R_{x_1 x_2}(\tau) = E[x_1(t)\, x_2(t - \tau)],$$

$$\hat{R}_{x_1 x_2}(\tau) = \frac{1}{T - \tau} \int_{\tau}^{T} x_1(t)\, x_2(t - \tau)\, dt,$$

- The time shift causing the peak is an estimate of the true delay D

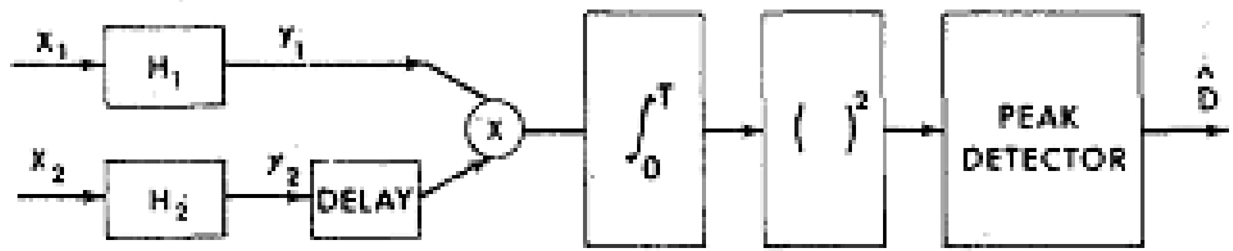

Fig. 1. Received waveforms filtered, delayed, multiplied, and integrated for a variety of delays until peak output is obtained.

$$R_{x_1x_2}(\tau) = \int_{-\infty}^{\infty} G_{x_1x_2}(f)\, e^{j2\pi f\tau}\, df.$$

$$R_{y_1y_2}^{(g)}(\tau) = \int_{-\infty}^{\infty} \psi_g(f)\, G_{x_1x_2}(f)\, e^{j2\pi f\tau}\, df,$$

where

$$\psi_g(f) = H_1(f)\, H_2^*(f)$$

Processor Interpretation

- Cross Correlation of x1 and x2

$$R_{x_1 x_2}(\tau) = \alpha R_{s_1 s_1}(\tau - D) + R_{n_1 n_2}(\tau).$$

Fourier Transform of above gives

$$G_{x_1 x_2}(f) = \alpha G_{s_1 s_1}(f)\, e^{-j2\pi f D} + G_{n_1 n_2}(f).$$

One interpretation is that the delta function has been spread by the Fourier transform of signal spectrum.

I.Roth Processor

- The weighting proposed by Roth

$$\psi_R(f) = \frac{1}{G_{x_1x_1}(f)}$$

- Cross Correlation is given by

$$\hat{R}^{(R)}_{y_1y_2}(\tau) = \int_{-\infty}^{\infty} \frac{\hat{G}_{x_1x_2}(f)}{G_{x_1x_1}(f)} e^{j2\pi f\tau}\, df.$$

$$R^{(R)}_{y_1y_2}(\tau) = \delta(\tau - D) \circledast \int_{-\infty}^{\infty} \frac{\alpha G_{s_1s_1}(f)}{\{G_{s_1s_1}(f) + G_{n_1n_1}(f)\}} \cdot e^{j2\pi f\tau}\, df.$$

- Desirable effect of suppressing those frequency regions where $G_{n_1n_1}(f)$ is large.

Smoothed Coherence Transform(SCOT)

- SCOT selects

$$\psi_S(f) = 1/\sqrt{G_{x_1x_1}(f)\,G_{x_2x_2}(f)}.$$

- This weighting gives the SCOT

$$\hat{R}^{(S)}_{y_1y_2}(\tau) = \int_{-\infty}^{\infty} \hat{\gamma}_{x_1x_2}(f)\, e^{j2\pi f\tau}\, df,$$

where

$$\hat{\gamma}_{x_1x_2}(f) \triangleq \frac{\hat{G}_{x_1x_2}(f)}{\sqrt{G_{x_1x_1}(f)\,G_{x_2x_2}(f)}}.$$

- When $G_{x_1x_1}(f) = G_{x_2x_2}(f)$, the SCOT is equivalent to the Roth processor.

The Phase Transform(PHAT)

- To avoid the spreading, PHAT uses weighting $\psi_p(f) = \frac{1}{|G_{x_1x_2}(f)|}$
- With uncorrelated noise i.e., $G_{n_1n_2}(f) = 0$
  Cross Correlation is given by $R^{(p)}_{y_1y_2}(\tau) = \delta(t - D)$.
  since $\frac{\hat{G}_{x_1x_2}(f)}{|G_{x_1x_2}(f)|} = e^{j\theta(f)} = e^{j2\pi fD}$
- For the model as in fig1 with uncorrelated noises, PHAT doesnot suffer the spreading as other processors.

Contd….

- Estimate of $R^{(p)}_{y_1 y_2}(\tau)$ will not be a delta function if $\hat{G}_{x_1 x_2}(f) \neq G_{x_1 x_2}(f), \ \theta(f) \neq 2\pi f D$
- If $G_{x_1 x_2}(f) = 0$ in some fequency band, then $\theta(f)$ is undefined in that band and the estimate of the phase is uniform $[-\pi, \pi]$ rad.
- So need for additional weight to compensate for the presence or absence of the signal.
- SCOT assigns weight according to signal and noise characteristics.

Eckart Filter

- Weighting is given by $\psi_E(f) = \frac{\alpha G_{s_1 s_1}(f)}{G_{n_1 n_1}(f) G_{n_2 n_2}(f)}$.
- This possesses some of the qualities of SCOT such as supressing frequency bands of high noise.
- This filter attaches zero weigh to bands where $G_{s_1 s_1}(f) = 0$PHAT .

The HT Processor(Hannon and Thomson)

- ML estimator selects as the estimate of delay the value of τ at which

$$R_{y_1y_2}^{(\mathrm{HT})}(\tau)=\int_{-\infty}^{\infty}\hat{G}_{x_1x_2}(f)\frac{1}{|G_{x_1x_2}(f)|}\cdot\frac{|\gamma_{12}(f)|^2}{[1-|\gamma_{12}(f)|^2]}e^{j2\pi f\tau}\,df$$

achieves peak.

- Weight is given by $\psi_{\mathrm{HT}}(f)=\frac{1}{|G_{x_1x_2}(f)|}\cdot\frac{|\gamma_{12}(f)|^2}{[1-|\gamma_{12}(f)|^2]}$,

Interpretation of Low SNR of ML Estimator

- Good delay estimation is most difficult in the case of low SNR.
- For Low SNR $\frac{G_{s_1 s_1}(f)}{G_{n_1 n_1}(f)} << 1$ and $\frac{G_{s_1 s_1}(f)}{G_{n_2 n_2}(f)} << 1,$

It follows that $\psi_{HT}(f) \cong \frac{G_{s_1 s_1}(f)}{G_{n_1 n_1}(f) G_{n_2 n_2}(f)} = \psi_E(f).$

- . $G_{n_1 n_1}(f) = G_{n_2 n_2}(f) = G_{nn}(f),$

$$\psi_{HT}(f) \cong \frac{G_{s_1 s_1}(f)}{G_{nn}(f)} [\psi_s(f)] = \left[\frac{G_{s_1 s_1}(f)}{G_{nn}(f)}\right]^2 \psi_p(f).$$

Conclusion

- HT processor has been shown to be an estimator time delay under usual conditions.
- Under Low SNR, the HT processor is equivalent to Eckart prefiltering and cross correlation.
- If the coherence is slowly changing as a function of time, the ML estimation is a cross correlator preceded by prefilters that vary with time.

Thank you