Term Paper Report

EE602

Array Processing Through Conventional Spatial Filtering

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INTRODUCTION

The subject of array processing is concerned with the extraction of information from signals collected using an array of sensors. These signals propagate spatially through a medium, for example, air or water, and the resulting wavefront is sampled by the sensor array. The information of interest in the signal may be either the content of the signal itself (communications) or the location of the source or reflection that produces the signal (radar and sonar). In either case, the sensor array data must be processed to extract this useful information.

We introduce the concept of beamforming, that is, the spatial discrimination or filtering of signals collected with a sensor array. We look at conventional, that is, nonadaptive, beamforming and touch upon many of the common considerations for an array that affect its performance, for example, element spacing, resolution, and sidelobe levels.

ARRAY FUNDAMENTALS

Single sensor system:
It is commonly found in communications and radar applications in which the signals are collected over a continuous spatial extent or aperture using a parabolic dish. The signals are reflected to the antenna in such a way that signals from the direction in which the dish is pointed are emphasized. The ability of a sensor to spatially discriminate, known as *directivity*, is governed by the shape and physical characteristics of its geometric structure.

Drawbacks:

1. Since the sensor relies on mechanical pointing for directivity, it can extract and track signals from only one direction at a time; it cannot look in several directions simultaneously.

2. Such a sensor cannot adapt its response, which would require physically changing the aperture, in order to reject potentially strong sources that may interfere with the extraction of the signals of interest

Multi array sensor system:
The sensor array signals are combined in such a way that a particular direction is emphasized. However, the direction in which the array is focused or pointed is almost independent of the orientation of the array. Therefore, the sensors can be combined in distinct, separate ways so as to emphasize different directions, all of which may contain signals of interest.

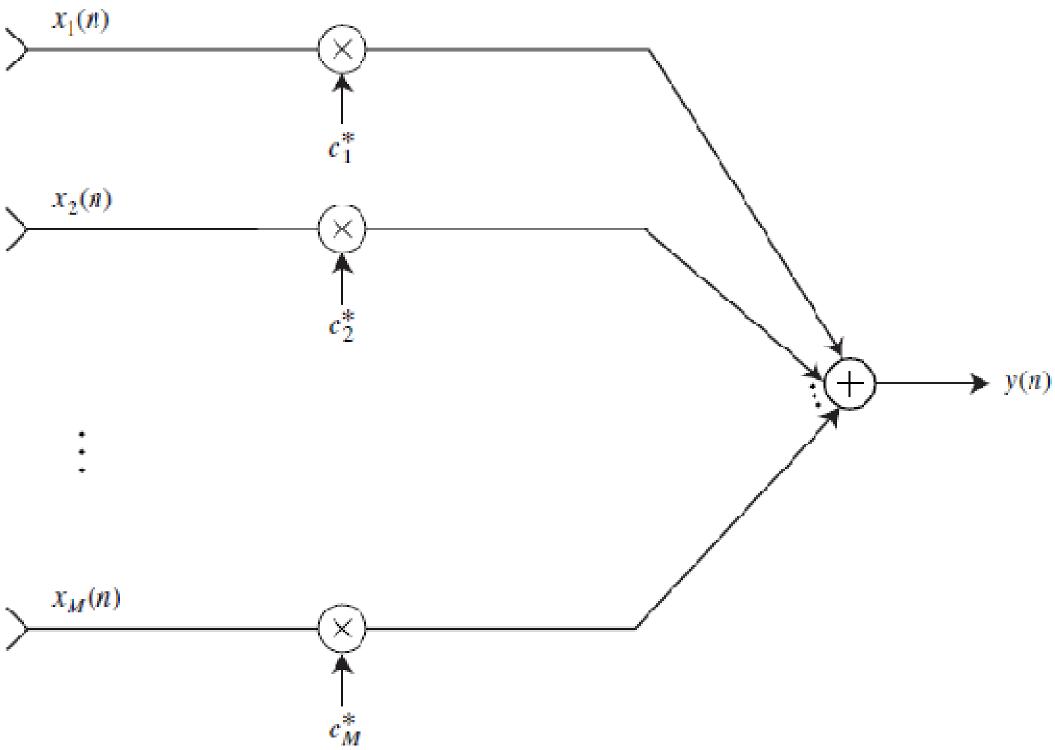
Advantages:

1. Since the various weighted summations of the sensors simply amount to processing the same data in different ways, these multiple sources can be extracted simultaneously.

2. Arrays have the ability to adjust the overall rejection level in certain directions to overcome strong interference sources.

Conventional Spatial Filtering: Beamforming

Here, we linearly combine the signals from all the sensors in a manner, that is, with a certain weighting, so as to examine signals arriving from a specific angle. This operation is known as *beamforming* because the weighting process emphasizes signals from a particular direction while attenuating those from other directions and can be thought of as casting or forming a beam. In this sense, a beamformer is a spatial filter; and in the case of a ULA, it has a direct analogy to an FIR frequency-selective filter for temporal signals Beamforming is commonly referred to as "electronic" steering since the weights are applied using electronic circuitry following the reception of the signal for the purpose of steering the array in a particular direction.



In its most general form, a beamformer produces its output by forming a weighted combination of signals from the *M* elements of the sensor array, that is,

$$y(n) = \sum_{m=1}^{M} c_m^* x_m(n) = \mathbf{c}^H \mathbf{x}(n)$$

$$\mathbf{c} = [c_1 \; c_2 \; \cdots \; c_M]^T$$

where **c** is the column vector of beamforming weights.

Beam response

A standard tool for analyzing the performance of a beamformer is the response for a given weight vector **c** as a function of angle φ, known as the *beam response*. This angular response is computed by applying the beamformer **c** to a set of array response vectors from all possible angles, that is, $-90^\circ \leq \varphi < 90^\circ$,

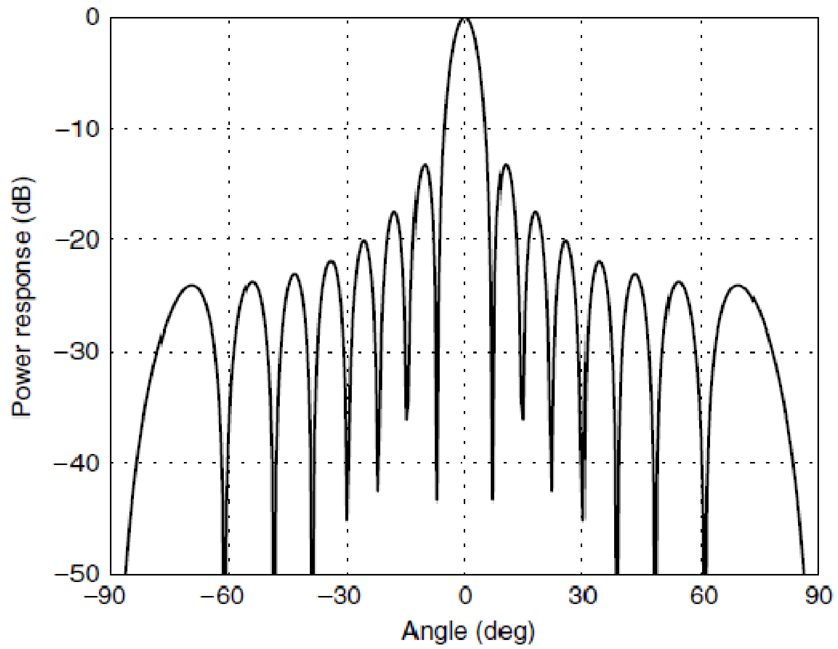
$$C(\phi) = \mathbf{c}^H \mathbf{v}(\phi)$$

Typically, in evaluating a beamformer, we look at the quantity $|C(\varphi)|^2$, which is known as the *beampattern*. Alternatively, the beampattern can be computed as a function of normalized spatial frequency *u.*

To compute the corresponding angles of the beampattern, we can simply convert spatial frequency to angle as

$$\phi = \arcsin \frac{\lambda}{d} u$$

A sample beampattern for a 16-element uniform array with uniform weighting ($c_m = 1/\sqrt{M}$) is shown in Figure as mentioned below. which is plotted on a logarithmic scale in decibels. The large mainlobe is centered at $\varphi = 0^\circ$, the direction in which the array is steered. Also notice the unusual sidelobe structure created by the nonlinear relationship between angle and spatial frequency in figure at angles away from broadside ($\varphi = 0^\circ$).



Steered Response:

Response of the array to a certain set of spatial signals impinging on array as we steer array to all possible angles. Since this operation corresponds to measuring the power as a function of spatial frequency or angle, the steered response might be better defined as the *spatial power spectrum*

$$R(\phi) = E\{|\mathbf{c}^H(\phi)\mathbf{x}(n)|^2\}$$

where the choice of the beamformer $\mathbf{c}(\varphi)$ determines the type of spatial spectrum, say, conventional or minimum-variance.

Output Signal to noise Ratio:

We now look at the signal-to-noise ratio (SNR) of the beamformer output and determine the improvement in SNR with respect to each element, known as the *beamforming gain*. Let us consider the signal model for a ULA, which consists of a signal of interest arriving from an angle φ_s and thermal sensor noise $\mathbf{w}(n)$. The beamformer or spatial filter $\mathbf{c}$ is applied to the array signal $\mathbf{x}(n)$ as

$$y(n) = \mathbf{c}^H \mathbf{x}(n) = \sqrt{M}\mathbf{c}^H \mathbf{v}(\phi_s)s(n) + \bar{w}(n)$$

where $w(n)$ = $c^H w(n)$ is the noise at the beamformer output and is also temporally uncorrelated.

The beamformer output power is

$$P_y = E\{|y(n)|^2\} = \mathbf{c}^H \mathbf{R}_x \mathbf{c}$$
$$\mathbf{R}_x = E\{\mathbf{x}(n)\mathbf{x}^H(n)\}$$

And the signal for the m^{th} element is given by

$$x_m(n) = e^{-j2\pi(m-1)u_s} s(n) + w_m(n)$$

where u_s is the normalized spatial frequency of the array signal produced by $s(n)$. The signal $s(n)$ is the signal of interest within a single sensor including the sensor response $H_m(F_c)$. Therefore, the signal-to-noise ratio in each element is given by

$$\text{SNR}_{\text{elem}} \triangleq \frac{\sigma_s^2}{\sigma_w^2} = \frac{|e^{-j2\pi(m-1)u_s} s(n)|^2}{E\{|w_m(n)|^2\}}$$

where $\sigma_s^2 = E\{|s(n)|2\}$ and $\sigma_w^2 = E\{|wm(n)|2\}$ are the element level signal and noise powers, respectively. Recall that the signal $s(n)$ has a deterministic amplitude and random phase. We assume that all the elements have equal noise power σ_w^2 so that the SNR does not vary from element to element. This SNR_{elem} is commonly referred to as the *element level SNR* or the *SNR per element*.

Now if we consider the signals at the output of the beamformer, the signal and noise powers are given by

$$P_s = E\{|\sqrt{M}[\mathbf{c}^H \mathbf{v}(\phi_s)]s(n)|^2\} = M\sigma_s^2|\mathbf{c}^H \mathbf{v}(\phi_s)|^2$$
$$P_n = E\{|\mathbf{c}^H \mathbf{w}(n)|^2\} = \mathbf{c}^H \mathbf{R}_n \mathbf{c} = \|\mathbf{c}\|^2 \sigma_w^2$$

because $R_n = \sigma_w^2 I$. Therefore, the resulting SNR at the beamformer output, known as the *array SNR*, is

$$\mathrm{SNR}_{\mathrm{array}} = \frac{P_s}{P_n} = \frac{M|\mathbf{c}^H \mathbf{v}(\phi_s)|^2}{\|\mathbf{c}\|^2} \frac{\sigma_s^2}{\sigma_w^2} = \frac{|\mathbf{c}^H \mathbf{v}(\phi_s)|^2}{\|\mathbf{c}\|^2} M\,\mathrm{SNR}_{\mathrm{elem}}$$

which is simply the product of the beamforming gain and the element level SNR. Thus, the *beamforming gain* is given by

$$G_{\mathrm{bf}} \triangleq \frac{\mathrm{SNR}_{\mathrm{array}}}{\mathrm{SNR}_{\mathrm{elem}}} = \frac{|\mathbf{c}^H \mathbf{v}(\phi_s)|^2}{|\mathbf{c}|^2} M$$

The beamforming gain is strictly a function of the angle of arrival φ_s of the desired signal, the beamforming weight vector **c**, and the number of sensors *M*.

Spatial Matched Filter:

The array signal model of a single signal, arriving from a direction φ_s , with sensor thermal noise

$$\mathbf{x}(n) = \sqrt{M}\mathbf{v}(\phi_s)s(n) + \mathbf{w}(n)$$
$$= [s(n)\ e^{-j2\pi u_s}s(n)\ \cdots\ e^{-j2\pi(M-1)u_s}s(n)]^T + \mathbf{w}(n)$$

where the components of the noise vector w*(n)* are uncorrelated and have power σ_w^2, that is, $E\{w(n)w^H(n)\} = \sigma_w^2 I$. The individual elements of the array contain the same signal *s(n)* with different phase shifts corresponding to the differences in propagation times between elements. Ideally, the signals from the *M* array sensors are added coherently, which requires that each of the relative phases be zero at the point of summation; that is, we add *s(n)* with a perfect replica of itself. Thus, we need a set of complex weights that results in a perfect phase alignment of all the sensor signals. The beamforming weight vector that phase-aligns a signal from direction φ_s at the different array elements is the *steering vector*, which is simply the array response vector in that direction, that is,

$$\mathbf{c}_{\text{mf}}(\phi_s) = \mathbf{v}(\phi_s)$$

The steering vector beamformer is also known as the *spatial matched filter*†since the steering vector is matched to the array response of signals impinging on the array from an angle φ_s . As a result, φ_s is known as the *look direction*. The use of the spatial matched filter is commonly referred to as *conventional beamforming*.

The output of the spatial matched filter is

$$y(n) = \mathbf{c}_{\text{mf}}^H(\phi_s)\mathbf{x}(n) = \mathbf{v}^H(\phi_s)\mathbf{x}(n)$$
$$= \frac{1}{\sqrt{M}}[1\ e^{j2\pi u_s}\ \cdots\ e^{j2\pi(M-1)u_s}]$$
$$\times \left\{ \begin{bmatrix} s(n) \\ e^{-j2\pi u_s}s(n) \\ \vdots \\ e^{-j2\pi(M-1)u_s}s(n) \end{bmatrix} + \mathbf{w}(n) \right\}$$
$$= \frac{1}{\sqrt{M}}[s(n) + s(n) + \cdots + s(n)] + \bar{w}(n)$$
$$= \sqrt{M}\, s(n) + \bar{w}(n)$$

Examining the array SNR of the spatial matched filter output, we obtain

$$\begin{aligned}\mathrm{SNR}_{\mathrm{array}} &= \frac{P_s}{P_n} = \frac{M\sigma_s^2}{E\{|\mathbf{v}^H(\phi_s)\mathbf{w}(n)|^2\}} \\ &= \frac{M\sigma_s^2}{\mathbf{v}^H(\phi_s)\mathbf{R}_n\mathbf{v}(\phi_s)} = M\frac{\sigma_s^2}{\sigma_w^2} = M\cdot\mathrm{SNR}_{\mathrm{elem}}\end{aligned}$$

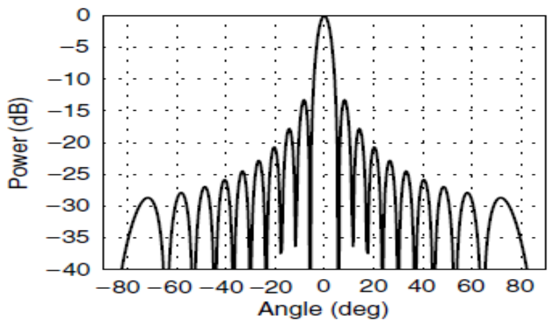
since $P_s = M\sigma_s^2$ and $R_n = \sigma_w^2 I$. Therefore, the beamforming gain is

$$G_f^b = M$$

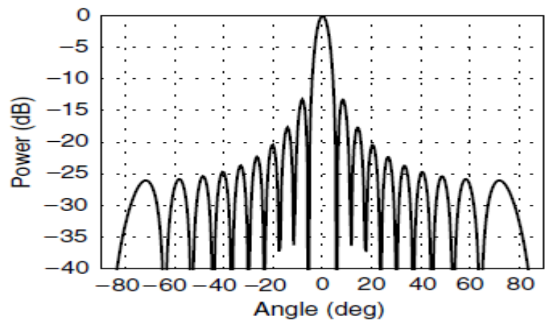
Element Spacing:

we show the beampatterns of spatial matched filters with $\varphi_s = 0^\circ$ for ULAs with element spacings of $\lambda/4$, $\lambda/2$, λ, and 2λ (equal-sized apertures of 10λ with 40, 20, 10, and 5 elements, respectively).

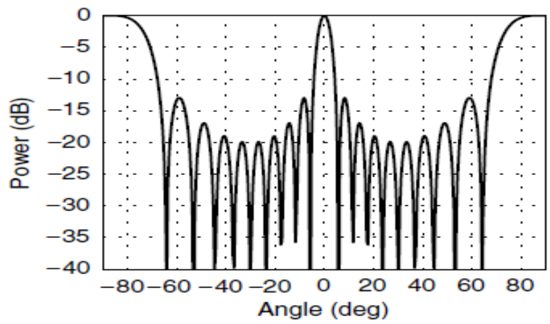
We note that the beampatterns for $\lambda/4$ and $\lambda/2$ spacing are identical with equal-sized mainlobes and the first sidelobe having a height of −13 dB. The oversampling for the array with an element spacing of $\lambda/4$ provides no additional information and therefore does not improve the beamformer response in terms of resolution. In the case of the undersampled arrays ($d = \lambda$ and 2λ), we see the same structure (beamwidth) around the look direction but also note the additional peaks in the beampattern (0 dB) at ±90◦ for $d = \lambda$ and in even closer for $d = 2\lambda$. These additional lobes in the beampattern are known as *grating lobes*. Grating lobes create spatial ambiguities; that is, signals incident on the array from the angle associated with a grating lobe will look just like signals from the direction of interest. The beamformer has no means of distinguishing signals from these various directions. In certain applications, grating lobes may be acceptable if it is determined that it is either impossible or very improbable to receive returns from these angles; for example, a communications satellite is unlikely to receive signals at angles other than those corresponding to the ground below. The benefit of the larger element spacing is that the resulting array has a larger aperture and thus better resolution.



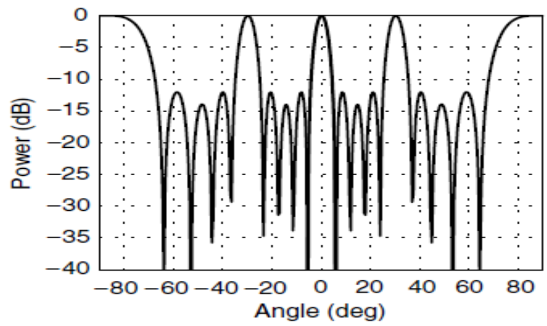
(a) $d = \lambda/4$



(b) $d = \lambda/2$



(c) $d = \lambda$



(d) $d = 2\lambda$

Array aperture and beamforming resolution:

The aperture is the finite area over which a sensor collects spatial energy. In the case of a ULA, the aperture is the distance between the first and last elements. The greater the aperture, the finer the resolution of the array, which is its ability to distinguish between closely spaced sources.

As a general rule of thumb, the −3-dB beamwidth for an array with an aperture length of L is quoted in radians as

$$\Delta\phi_{3\,\mathrm{dB}} \approx \frac{\lambda}{L}$$

improved resolution results in better angle estimation capabilities and increasing the aperture yields better resolution, with a factor-of-2 improvement for each of the successive twofold increases in aperture length.

Tapered Beamforming:

The spatial matched filter would be perfectly sufficient if the only signal present, aside from the sensor thermal noise, were the signal of interest. However, in many instances we must contend with other, undesired signals that hinder our ability to extract the signal of interest. These signals may also be spatially propagating at the same frequency as the operating frequency of the array. We refer to such signals as *interference*. These signals may be present due to hostile adversaries that are attempting to prevent us from receiving the signal of interest, for example, jammers in radar or communications; or they might be incidental signals that are present in our current operating environment, such as transmissions by other users in a communications system or radar clutter.

Consider the ULA signal model from , but now including an interference signal i(n) made up of P interference sources

$$\mathbf{x}(n) = \mathbf{s}(n) + \mathbf{i}(n) + \mathbf{w}(n) = \sqrt{M}\mathbf{v}(\phi_s)s(n) + \sqrt{M}\sum_{p=1}^{P}\mathbf{v}(\phi_p)i_p(n) + \mathbf{w}(n)$$

where $\mathbf{v}(\varphi_p)$ and $i_p(n)$ are the array response vector and actual signal due to the pth interferer, respectively. If we have a ULA with $\lambda/2$ element spacing, the beampattern of the spatial matched filter, as shown in Figure, may have sidelobes that are high enough to pass these interferers through the beamformer with a high enough gain to prevent us from observing the desired signal. For this array, if an interfering source were present at $\varphi = 20^\circ$ with a power of 40 dB, the power of the interference at the output of the spatial matched filter would be 20 dB because the sidelobe level at $\varphi = 20^\circ$ is only −20 dB. Therefore, if we were trying to receive a weaker signal from $\varphi s = 0^\circ$, we would be unable to extract it because of sidelobe leakage from this interferer.

The spatial matched filter has weights all with a magnitude equal to $1/\sqrt{M}$. The look direction is determined by a linear phase shift across the weights of the spatial matched filter. However, the sidelobe levels can be further reduced by tapering the magnitudes of the spatial matched filter. To this end, we employ a tapering vector $\mathbf{t}$ that is applied to the spatial matched filter to realize a low sidelobe level beamformer

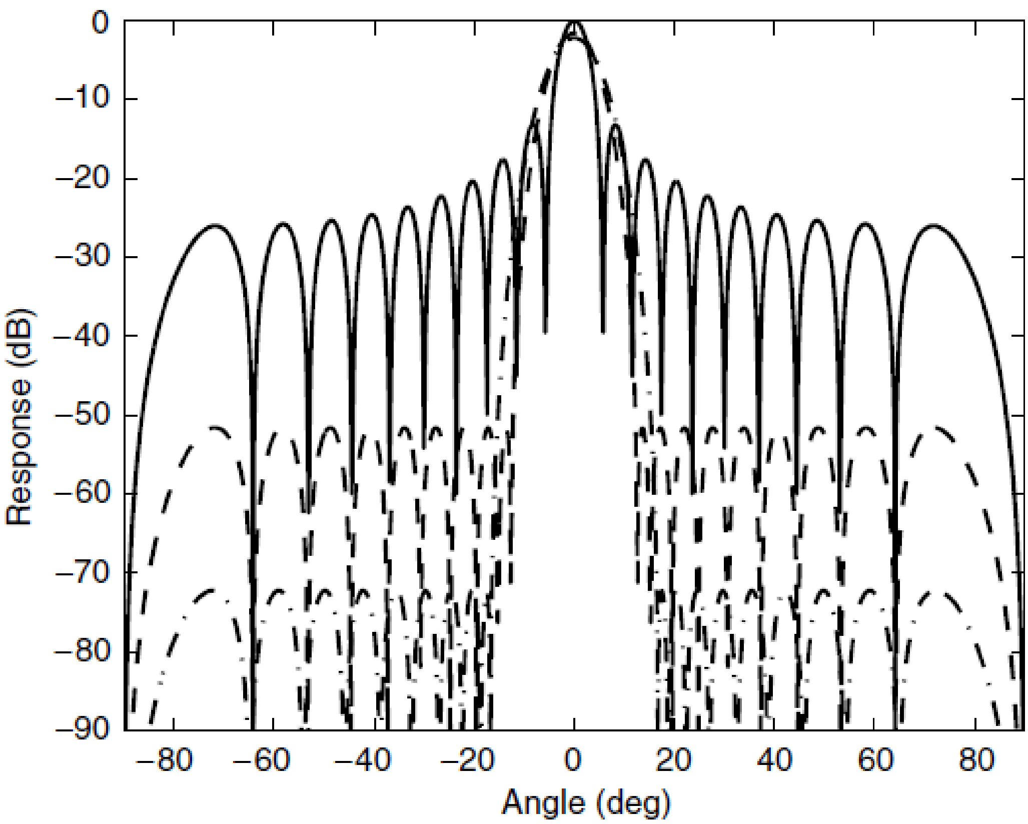
$$\mathbf{c}_{\text{tbf}}(\phi_s) = \mathbf{t} \odot \mathbf{c}_{\text{mf}}(\phi_s)$$

We refer to this beamformer as the *tapered beamformer*.

The determination of a taper can be thought of as the design of the desired beamformer where c_{mf} simply determines the desired angle. The weight vector of the spatial matched filter from has unit norm. Similarly, the tapered beamformer $\mathbf{c}_{tbf}$ is normalized so that

$$\mathbf{c}_{tbf}^H(\phi_s)\mathbf{c}_{tbf}(\phi_s) = 1$$

This taper produces a constant sidelobe level (equiripples in the stopband in spectral estimation), which is often a desirable attribute of a beamformer. The best taper choice is driven by the actual application. The beampatterns of the ULA are used again, but this time the beampatterns of tapered beamformers are also shown in Figure.The sidelobe levels of the tapers were chosen to be −50 and −70 dB.† The same 40-dB interferer would have been reduced to −10 and −30 dB at the beamformer output, respectively.



Notice that the mainlobes of the beampatterns in Figure are much broader for the tapered beamformers. The consequence is a loss in resolution that becomes more pronounced as the tapering is increased to achieve lower sidelobe levels. Note that the elements on the ends of the array are given less weighting as the tapering level is increased. The tapered array in effect deemphasizes these end elements while emphasizing the center elements. Therefore, the loss in resolution for a tapered beamformer might be interpreted as a loss in the effective aperture of the array imparted by the tapering vector.