Array processing

EE 602

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Subject:

- **Is concerned with the extraction of information from signals collected using an array of sensors**

- **The information of interest in the signal may be either the content of the signal or the location of the source or reflection that produces the signal**

Assumption:

- **We concentrate on uniform linear arrays**

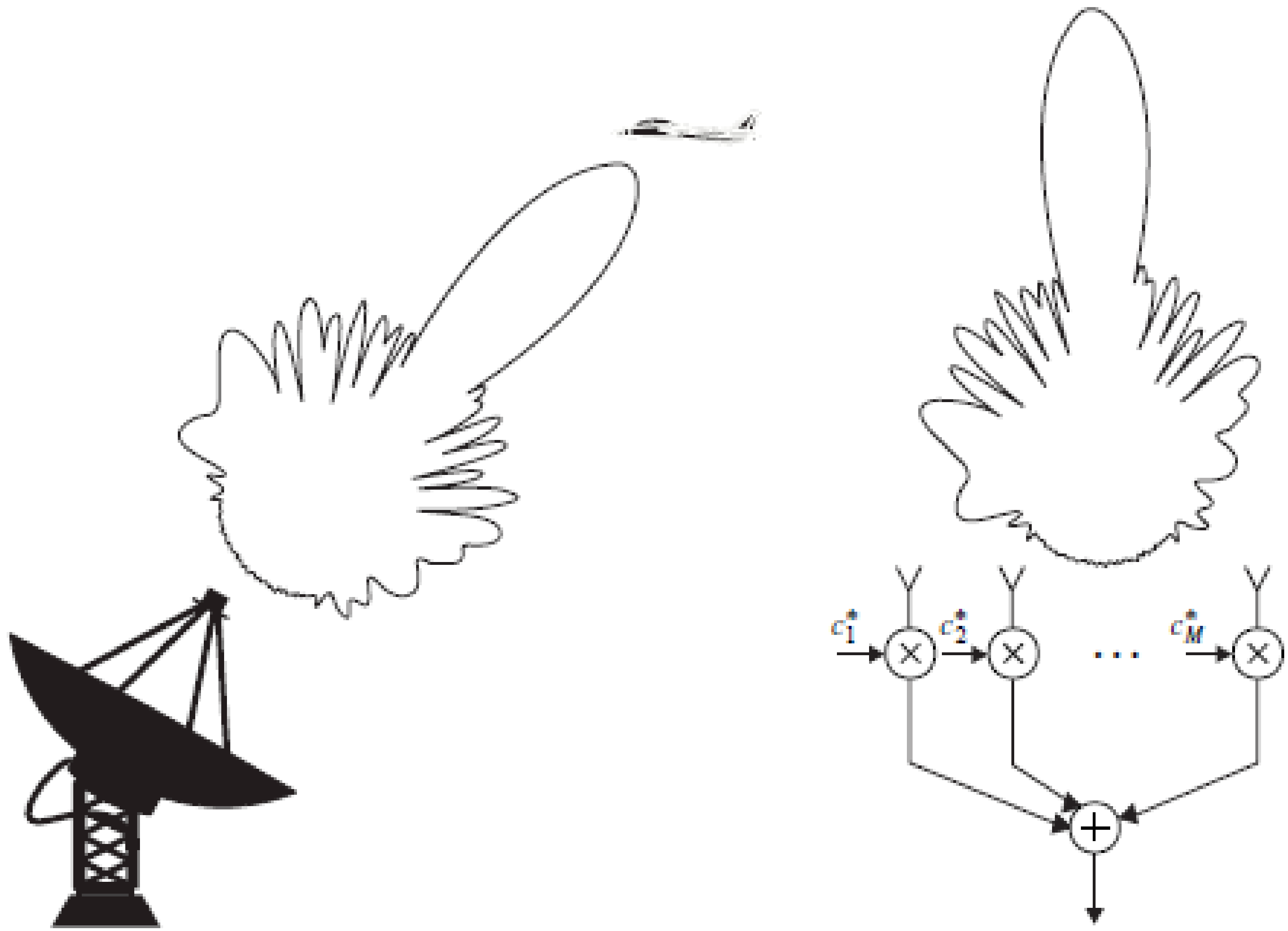
Need for an Array :

Drawbacks of single sensor system

1. **Mainly it depends on mechanical pointing for directivity, therefore it can extract and track signal only from one direction only**
2. **it cannot adopt its response in order to reject potentially strong source**

Hence by using an array of sensors we can overcome the drawbacks of single sensor system because of

1. **The sensor array signal are combined in such away that a particular direction is emphasized.**
2. **arrays have ability to adjust the overall rehection level in certain directions to overcome strong interference sources.**



(*a*) Parabolic dish antenna (continuous aperture)

(*b*) Sensor array antenna (discrete spatial aperture)

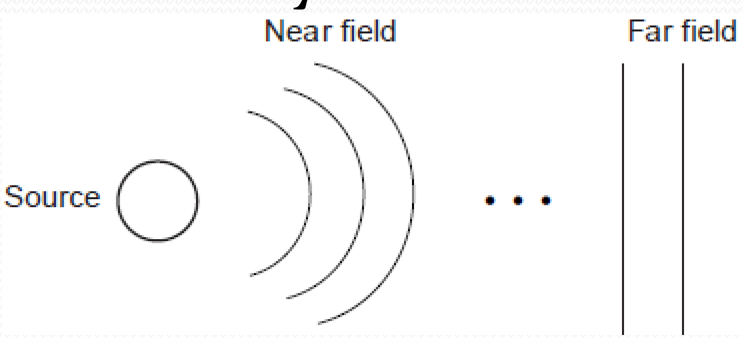
Examples for single sensor and array of sensors

We are going to study about

- **Spatial signals**
- **Modulation and demodulation**
- **Array signal model**
- **The sensor array: Spatially sampling**

Assumptions :

- **Lossless medium**
- **Non dispersive propagating medium**
- **Propagating signals are assumed to be produced by a point source**
- **Source is assumed to be in far field**
- **Multiple sources are treated through superposition of the various spatial signals at the sensor array**

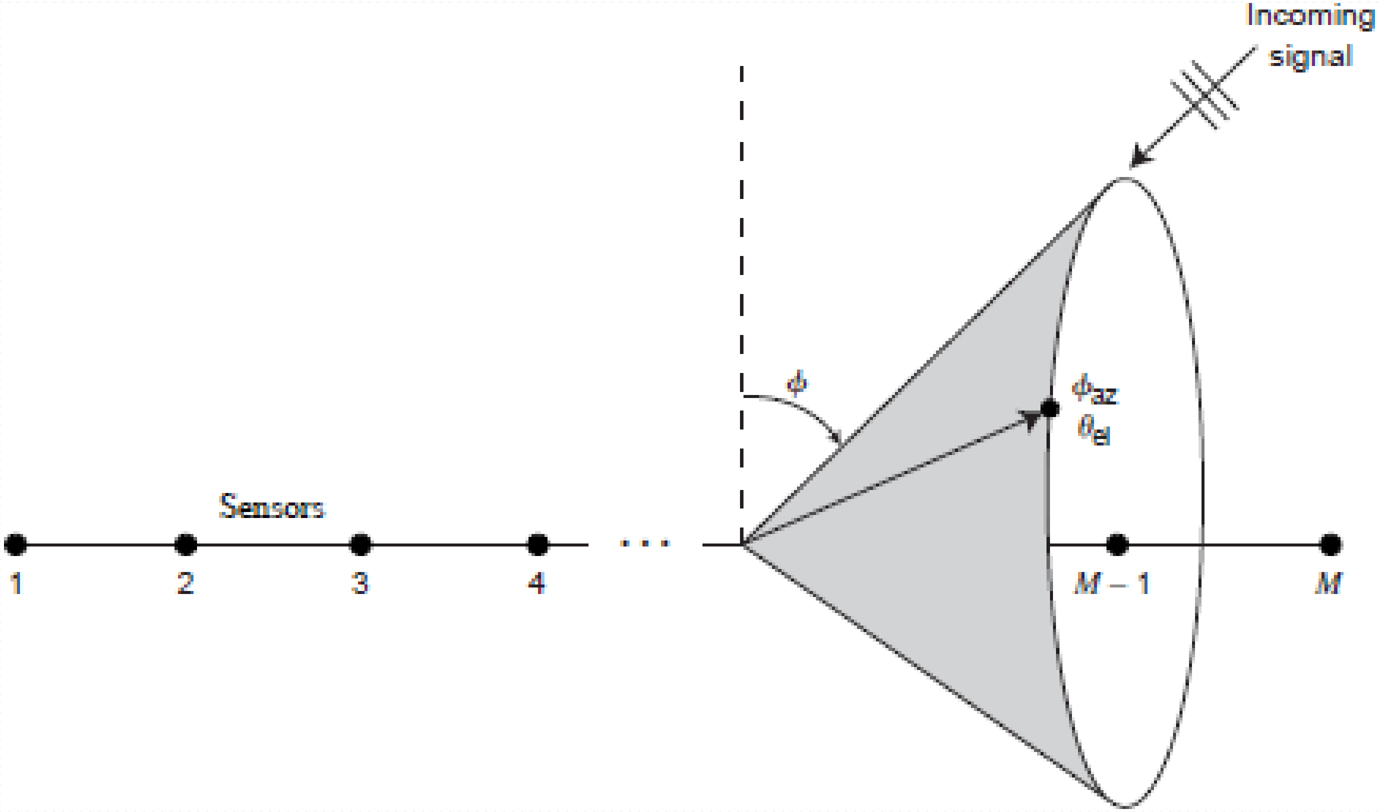


Plane wave approximation in the far field of the source.

Uniform linear array:

- **The array consists a series of elements located on a line with uniform spacing .**
- **The differences in distance between the sensors determine the relative delays in arrival of the plane wave, let $\|\mathbf{r}\|$ be the distance from which wave is coming and $\varnothing_{az}$ *and* θ_{el} are azimuthal and elevation angle respectively .**
- **Then the distance between neighboring elements for a plane wave**

$$d_x = \|\mathbf{r}\| \sin\phi_{\mathrm{az}} \cos\theta_{\mathrm{el}}$$



Cone angle ambiguity surface for a uniform linear array.

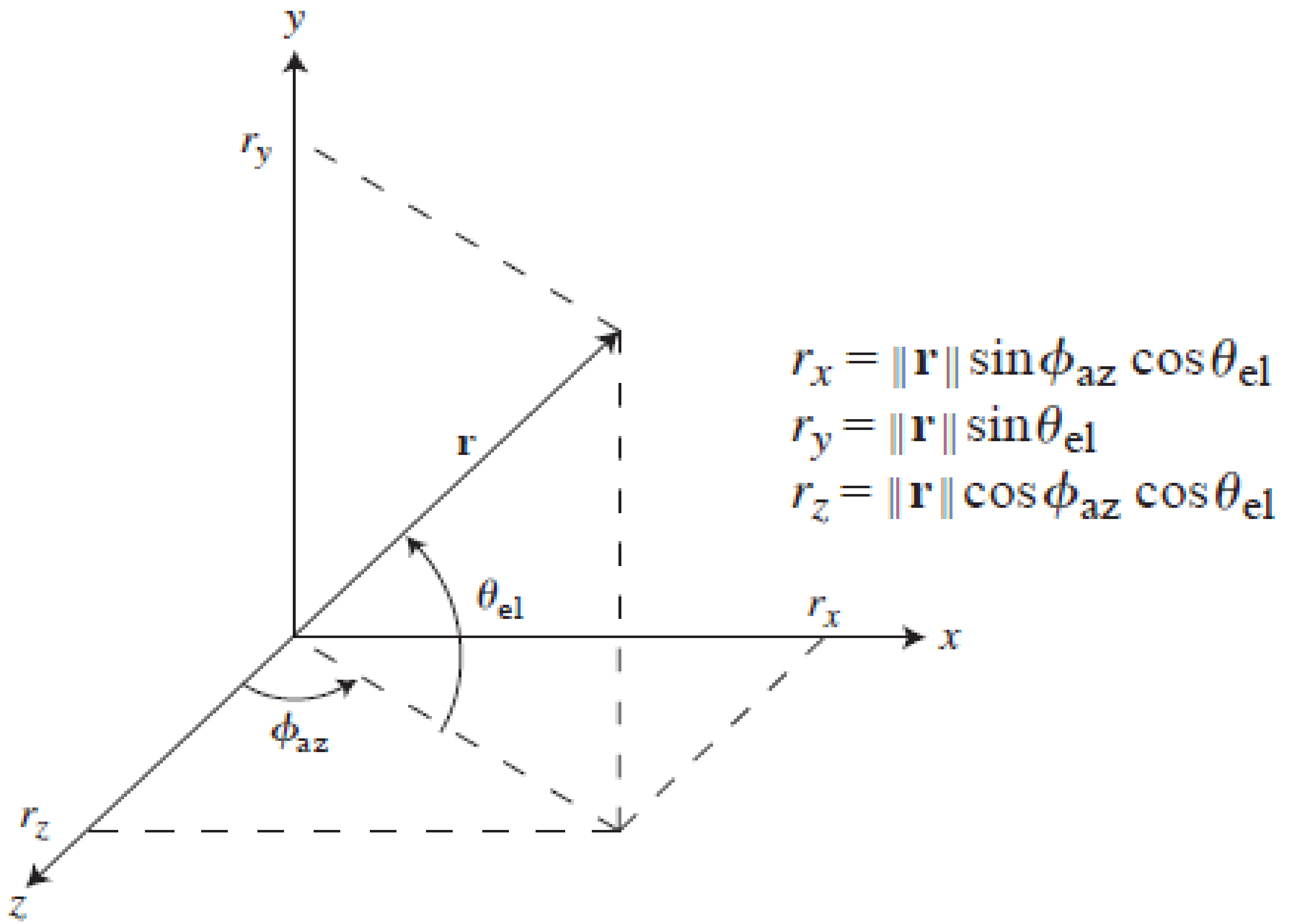
Arrival of the signal at an ∅ angle of on a uniform linear array.

Spatial signals:

- **Spatial signals are signals that propagate through space**

A spatial signal at a point specified by the vector r can be represented either in Cartesian coordinates *(x, y, z) or in spherical coordinates (R, $\varnothing_{az}$, θ_{el}) as shown in below Figure*.

***Here, R = ||r|| represents range or the distance from the origin, and $\varnothing_{az}$ and θ_{el} are the azimuth* and elevation angles, respectively**



Three dimensional space describing azimuth ,elevation and range

The propagation of a signal is governed by the solution to the wave equation

A propagating wave emanating from a source located at $\mathbf{r}_o$, is a single frequency wave given by

$$s(t, \mathbf{r}) = \frac{A}{\|\mathbf{r} - \mathbf{r}_0\|^2} e^{j2\pi F_c \left(t - \frac{\|\mathbf{r}-\mathbf{r}_0\|}{c}\right)}$$

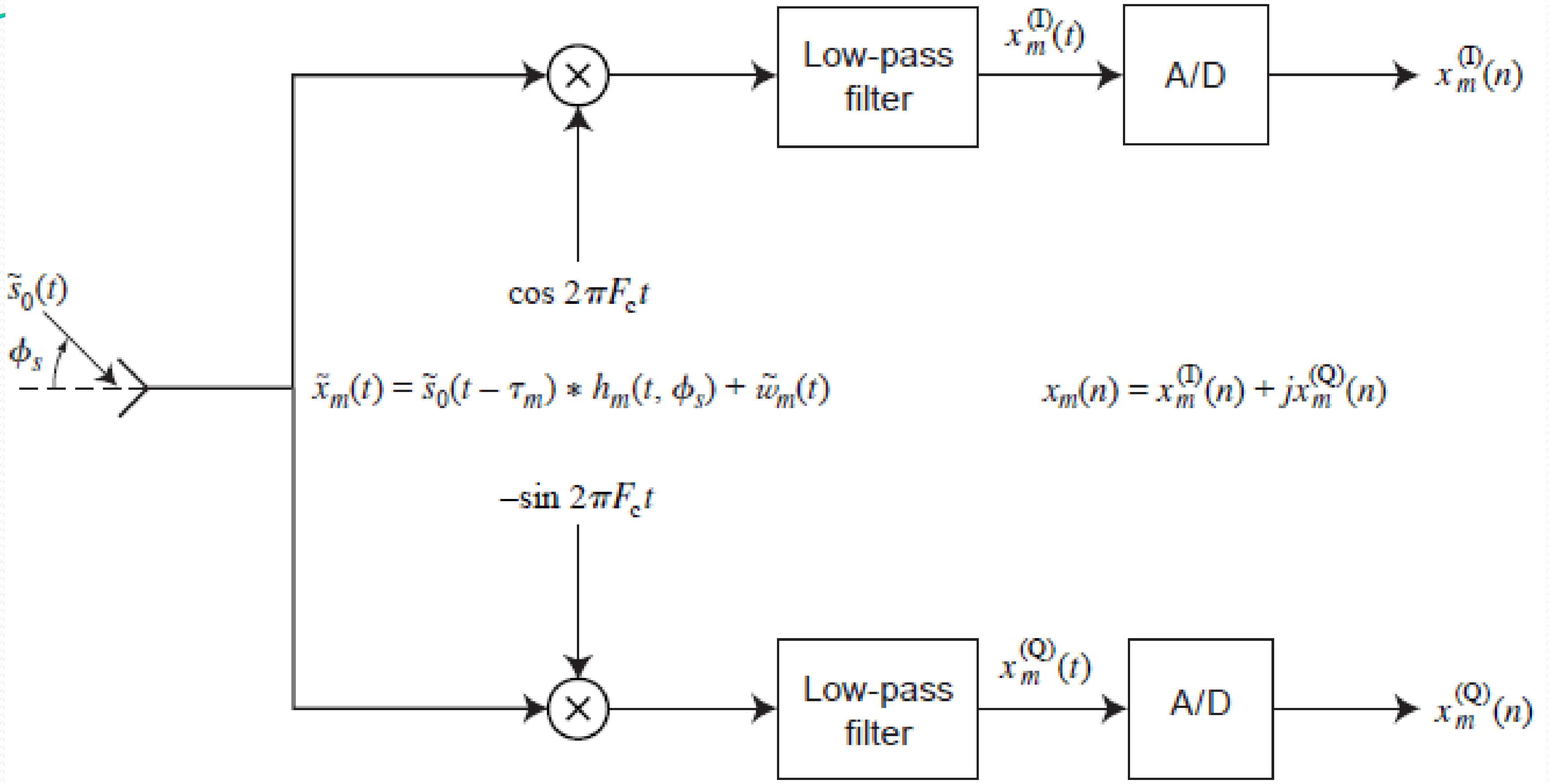
A **is the complex amplitude, F_c is the carrier frequency of the wave and c is the speed of propagation of wave**

Modulation and demodulation

- The process of generating the signal $\tilde{s}_0(t)$ from So(t) in order to transmit the information, is accomplished by mixing the signal So(t) with the carrier waveform cos $2\Pi F_c t$ in an operation known as modulation.

$$\tilde{s}_0(t) = s_0(t)\cos 2\pi F_c t = \frac{1}{2}s_0(t)(e^{j2\pi F_c t} + e^{-j2\pi F_c t})$$

- Upon reception of signal $\tilde{s}_0(t)$.the signal is mixed back to baseband in an operation known as demodulation and the process is shown in below block diagram



Block diagram of propagating signal arriving at a sensor with a receiver.

Array signal model

- A model for a single spatial signal in noise received by a ULA from an angle $\varnothing_s$
- $\tilde{x}_m(t)$ is the continuous-time signal in the m^{th} sensor containing both received carrier modulated signals and thermal noise . $x_m(t)$ is obtained by demodulating $\tilde{x}_m(t)$ to baseband and low pass filtering to the receiver

The signal received by mth sensor with a delay t_m

$$\tilde{x}_m(t) = h_m(t,\ \phi_s) * \tilde{s}_0(t - \tau_m) + \tilde{w}_m(t)$$

Where $h_m(t,\varnothing_s)$ is the impulse response of the m_{th} sensor as a function of both time and angle, and $\tilde{w}_m(t)$ is the sensor noise

The sensor received signal in frequency domain as

$$\tilde{X}_m(F) = H_m(F,\phi_s)\tilde{S}_0(F)e^{-j2\pi F\tau_m} + \tilde{W}_m(F)$$
$$= H_m(F,\phi_s)[S_0(F-F_c) + S_0^*(-F-F_c)]e^{-j2\pi F\tau_m} + \tilde{W}_m(F)$$

By demodulation and ideal low-pass filtering ,the spectrum of the signal is

$$X_m(F) = H_m(F+F_c,\phi_s)S_0(F)e^{-j2\pi(F+F_c)\tau_m} + W_m(F)$$

Assumptions :

- Narrow band (i.e TBWP << 1)
- Response of the sensor is constant across the bandwidth of the receiver $H_m(F+F_c,\phi_s) = H_m(F_c,\phi_s)$

And hence the response simplifies to

$$X_m(F) = H_m(F_c,\phi_s)S_0(F)e^{-j2\pi F_c\tau_m} + W_m(F)$$

The discrete-time signal model is obtained by sampling the inverse Fourier transform of $X_m(F)$

$$x_m(n) = H_m(F_c, \phi_s) s_0(n) e^{-j2\pi F_c \tau_m} + w_m(n)$$

Therefore the full array discrete-time signal model as

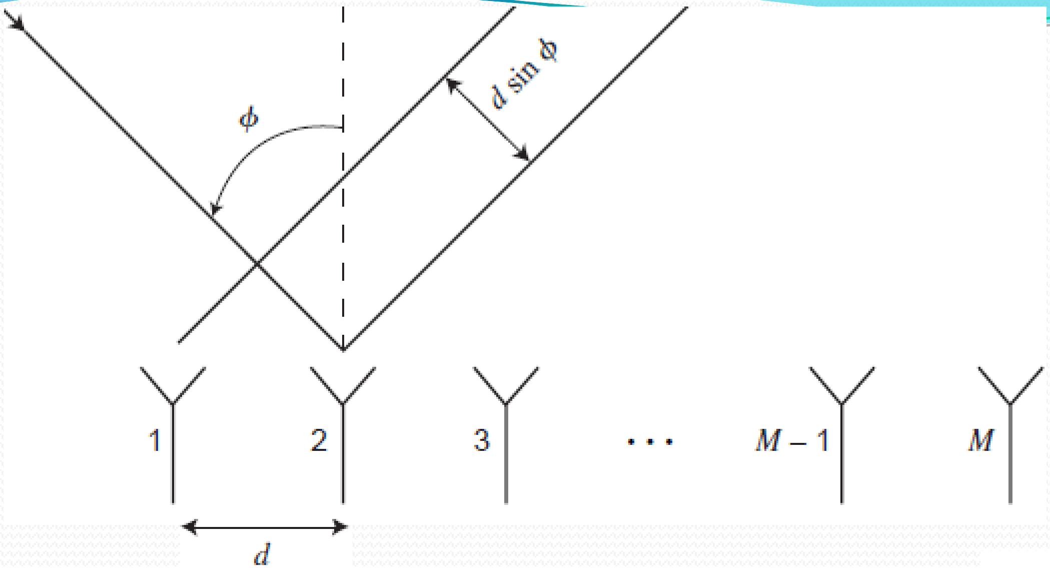
$$\mathbf{x}(n) = \sqrt{M}\, \mathbf{v}(\phi_s) s(n) + \mathbf{w}(n)$$

where The signal term $s(n) = H(F_c) s_0(n)$

and $$\mathbf{v}(\phi) = \frac{1}{\sqrt{M}} [1 \; e^{-j2\pi F_c \tau_2(\phi)} \; \ldots \; e^{-j2\pi F_c \tau_M(\phi)}]^T$$

is the array response vector

Plane wave impinging on a uniform linear array



The delay between two successive sensors is $\tau(\phi) = \dfrac{d \sin \phi}{c}$

The delay to the mth element with respect to the first element in the array is

$$\tau_m(\phi) = (m-1)\frac{d \sin \phi}{c}$$

Hence array response vector for a ULA is

$$\mathbf{v}(\phi) = \frac{1}{\sqrt{M}}[1 \; e^{-j2\pi[(d\sin\phi)/\lambda]} \; \ldots \; e^{-j2\pi[(d\sin\phi)/\lambda](M-1)}]^T$$

The sensor array: spatial sampling

- **A mechanism for spatially sampling wavefront propagating at a certain operating frequency**
- **Sampling frequency must be high enough so as not to create spatial ambiguities**
- **Spatially sampling frequency $U_s = 1/d$**

d = Sampling period

- **For a spatially propagating signal ,spatial frequency is given by**

$$U = \frac{\sin \phi}{\lambda}$$

Normalised spatial frequency is given by

$$u \triangleq \frac{U}{U_s} = \frac{d \sin \phi}{\lambda}$$

Therefore the array response vector in terms of normalised spatial frequency as

$$\mathbf{v}(\phi) = \mathbf{v}(u) = \frac{1}{\sqrt{M}} [1 \; e^{-j2\pi u} \; \cdots \; e^{-j2\pi u(M-1)}]^T$$

Which is a Vandermonde vector

- **Requirements on the spatial sampling frequency to avoid aliasing.**
- **Normalised frequencies are unambiguous for $-1/2 \leq u \leq 1/2$ and the full range of possible unambiguous angles is $-90 \leq \varnothing \leq 90$.**
- **The sensor spacing must be $d \leq \lambda/2$ to prevent spatial ambiguities.**

Thank you

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