A TERM PAPER REPORT
On

Array Processing
(Introduction)

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Object:

Is concerned with the extraction of information from signals collected using an array of sensors .The information of interest in the signal may be either the content of the signal or the location of the source or reflection that produces the signal. The methods used are extensions of the statistical and adaptive signal processing techniques discussed in previous chapters, such as spectral estimation and optimum and adaptive filtering, extended to sensor array applications. In this paper we concentrate on introduction of array processing only.

Introduction:

Need for an Array:

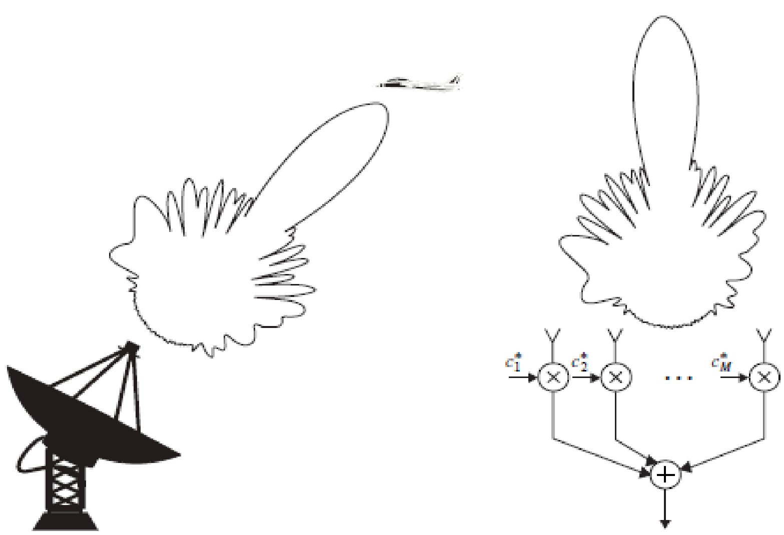
we deal with the signals in the presence of other, undesired signals. we can choose to focus on signals from a particular direction. This task can be accomplished by using a single sensor, provided that it has the ability to spatially discriminate; that is, it passes signals from certain directions while rejecting those from other directions the ability of a sensor to spatially discriminate, known as directivity.

Drawbacks of single sensor system

1. Mainly it depends on mechanical pointing for directivity, therefore it can extract and track signal only from one direction only
2. it cannot adopt its response in order to reject potentially strong source

Hence by using an array of sensors we can overcome the drawbacks of single sensor system because of the sensor array signal are combined in such away that a particular direction is emphasized.

Arrays have ability to adjust the overall rejection level in certain directions to overcome strong interference sources.

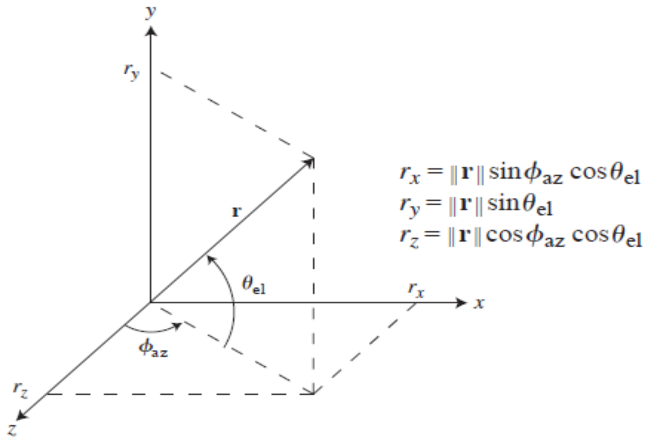


(*a*) Parabolic dish antenna (continuous aperture)

(*b*) Sensor array antenna (discrete spatial aperture)

Spatial signals:

spatial signals are signals that propagate through space. These signals originate from a source, travel through a propagation medium and arrive at an array of sensors that spatially samples the waveform. Since space is three-dimensional, a spatial signal at a point specified by the vector **r** can be represented either in Cartesian coordinates *(x, y, z)* or in spherical coordinates *(R, φ*az*, θ*el*)* as shown in Figure



Three dimensional space describing azimuth, elevation and range The propagation of a signal is governed by the solution to the wave equation A propagating wave emanating from a source located at r0, is a single frequency wave given by

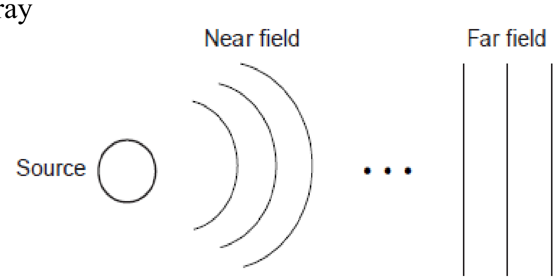
$$s(t, \mathbf{r}) = \frac{A}{\|\mathbf{r} - \mathbf{r}_0\|^2} e^{j2\pi F_c\left(t - \frac{\|\mathbf{r}-\mathbf{r}_0\|}{c}\right)}$$

where *A* is the complex amplitude, *F*c is the carrier frequency of the wave, and *c* is the speed of propagation of the wave

Assumptions:

1) Lossless medium
2) Non dispersive propagating medium
3) Propagating signals are assumed to be produced by a point source
4) Source is assumed to be in far field

Multiple sources are treated through superposition of the various spatial signals at the sensor array

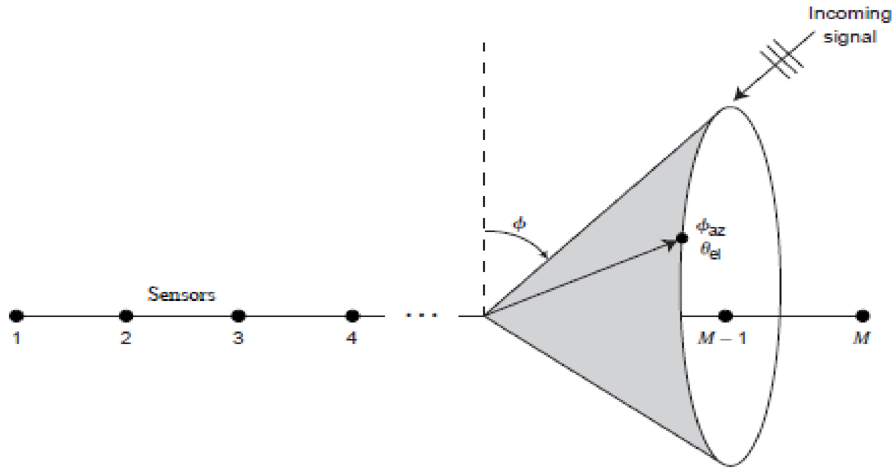


Uniform linear array:

The array consists a series of elements located on a line with uniform spacing. The differences in distance between the sensors determine the relative delays in arrival of the plane wave, let **r** be the distance from which wave is coming and *φ*az *and θel* are azimuthal and elevation angle respectively .

Then the distance between neighbouring elements for a plane wave

$$d_x = \|\mathbf{r}\| \sin\phi_{az} \cos\theta_{el}$$



Cone angle ambiguity surface for a uniform linear array

Arrival of the signal at an angle of φ on a uniform linear array. If we consider the entire three-dimensional space, we note that equivalent delays are produced by any signal arriving from a cone about the ULA as shown in above figure.

Modulation and demodulation:

The process of generating the signal $\tilde{s}_0(t)$ from $s_0(t)$ in order to transmit this information is accomplished by mixing the signal $s_0(t)$ with the carrier waveform $\cos 2\pi F_c t$ in an operation known as *modulation*.

$$\tilde{s}_0(t) = s_0(t) \cos 2\pi F_c t = \frac{1}{2} s_0(t)(e^{j2\pi F_c t} + e^{-j2\pi F_c t})$$

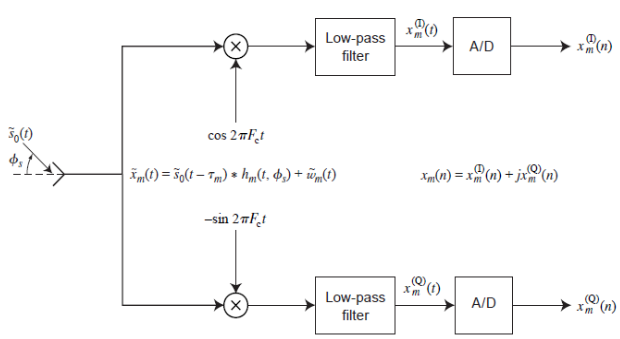
where we say that the signal $s_0(t)$ is carried by the propagating waveform $\cos 2\pi F_c t$.

The reception of spatially propagating signals with a sensor is only the beginning of the process of forming digital samples for both the in-phase and quadrature components of the sensor signal. Upon reception of the signal $\tilde{s}_0(t)$, the signal is mixed back to baseband in an operation known as *demodulation*. Included in the *m*th sensor signal is thermal noise due to the electronics of the sensor $w_m(t)$.

$$\tilde{x}_m(t) = \tilde{s}_0(t) * h_m(t, \varphi_s) + \tilde{w}_m(t)$$

Where $h_m(t, \varphi_s)$ is the combined temporal and spatial impulse response of the *m*th sensor. The angle φ_s is the direction from which $\tilde{s}_0(t)$ was received.

In The demodulation process involves multiplying the received signal by $\cos 2\pi F_c t$ and $-\sin 2\pi F_c t$ to form both the in-phase and quadrature channels, following demodulation, the signals in each channel are passed through a low-pass filter to remove any high-frequency components.



Block diagram of propagating signal arriving at a sensor with a receiver

Array signal model:

A model for a single spatial signal in noise received by a ULA from an angle φs is the continuous-time signal in the mth sensor containing both received carrier modulated signals and thermal noise . xm(t) is obtained by demodulating $\tilde{x}_m(t)$ to baseband and low pass filtering to the receive

$$\tilde{x}_m(t) = h_m(t, \ \phi_s) * \tilde{s}_0(t - \tau_m) + \tilde{w}_m(t)$$

Where hm (t, φs) is the impulse response of the mth sensor as a function of both time and angle, and $\tilde{w}_m(t)$ is the sensor noise

The sensor received signal in frequency domain as

$$\begin{aligned}\tilde{X}_m(F) &= H_m(F, \phi_s)\tilde{S}_0(F)e^{-j2\pi F\tau_m} + \tilde{W}_m(F) \\ &= H_m(F, \phi_s)[S_0(F - F_c) + S_0^*(-F - F_c)]e^{-j2\pi F\tau_m} + \tilde{W}_m(F)\end{aligned}$$

By demodulation and ideal low-pass filtering, the spectrum of the signal is

$$X_m(F) = H_m(F + F_c, \phi_s)S_0(F)e^{-j2\pi(F+F_c)\tau_m} + W_m(F)$$

Assumptions:

Narrow band (i.e. TBWP << 1)

Response of the sensor is constant across the bandwidth of the receiver

$$H_m(F + F_c, \phi_s) = H_m(F_c, \phi_s)$$

And hence the response simplifies to

$$X_m(F) = H_m(F_c, \phi_s)S_0(F)e^{-j2\pi F_c\tau_m} + W_m(F)$$

The discrete-time signal model is obtained by sampling the inverse Fourier transform of $X_m(F)$

$$x_m(n) = H_m(F_c, \phi_s)s_0(n)e^{-j2\pi F_c\tau_m} + w_m(n)$$

Therefore the full array discrete-time signal model as

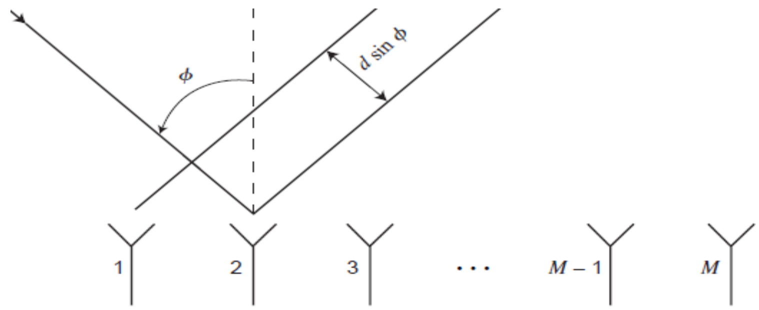
Where the signal term $s(n) = H(F_c)s_0(n)$

And

$$\mathbf{x}(n) = \sqrt{M}\,\mathbf{v}(\phi_s)s(n) + \mathbf{w}(n)$$

$$\mathbf{v}(\phi) = \frac{1}{\sqrt{M}}[1 \ e^{-j2\pi F_c\tau_2(\phi)} \ \dots \ e^{-j2\pi F_c\tau_M(\phi)}]^T$$

is the array response vector



Plane wave impinging on a uniform linear array

The delay between two successive sensors is

$$\tau(\phi) = \frac{d \sin \phi}{c}$$

The delay to the mth element with respect to the first element in the array is

$$\tau_m(\phi) = (m-1)\frac{d \sin \phi}{c}$$

Hence array response vector for a ULA is

$$\mathbf{v}(\phi) = \frac{1}{\sqrt{M}}[1 \; e^{-j2\pi[(d \sin \phi)/\lambda]} \; \dots \; e^{-j2\pi[(d \sin \phi)/\lambda](M-1)}]^T$$

The sensor array: spatial sampling

A mechanism for spatially sampling wave front propagating at a certain operating frequency

Sampling frequency must be high enough so as not to create spatial ambiguities , Similar to temporal sampling, the sensor array provides discrete (spatially sampled) data that can be used without loss of information, provided certain conditions are met. The sampling frequency must be high enough so as not to create spatial ambiguities or, in other words, to avoid spatial aliasing

Spatially sampling frequency Us = 1/d

d = Sampling period

For a spatially propagating signal, spatial frequency is given by

$$U = \frac{\sin \phi}{\lambda}$$

Normalised spatial frequency is given by

$$u \triangleq \frac{U}{U_s} = \frac{d \sin \phi}{\lambda}$$

Therefore the array response vector in terms of normalised spatial frequency as

$$\mathbf{v}(\phi) = \mathbf{v}(u) = \frac{1}{\sqrt{M}}[1 \; e^{-j2\pi u} \; \dots \; e^{-j2\pi u(M-1)}]^T$$

Which is a Vandermonde vector, a vector whose
Elements are successive integer powers of the same number, in this case $e{-}j2\pi u$. The interelement spacing d is simply the spatial sampling interval, which is the inverse of the sampling frequency. There are certain requirements on the spatial sampling frequency to avoid aliasing.
Since normalized frequencies are unambiguous for $-1/2 \leq u < ½$ and the full range of possible unambiguous angles is $-90^\circ \leq \varphi \leq 90^\circ$,

Therefore the required condition is
the sensor spacing must be $d \leq \lambda / 2$ to prevent spatial ambiguities.

Reference:

tatistical and adaptive signal processing by Stephen M Kogon.