Tutorial No. 2

1. The data $\{x[0], x[1], \ldots., x[N-1]\}$ are observed where the $x[n]'s$ are independent and identically distributed (IID) as $N\,[0, \sigma^2]$. We wish to estimate the variance σ^2 as

$$\widehat{\sigma^2} = \frac{1}{N}\sum_{n=0}^{N-1} x^2[n]$$

Is this an unbiased estimator? Find the variance of $\widehat{\sigma^2}$ and examine what happens as N -> ∞.

2. This problem illustrates what happens to an unbiased estimator when it undergoes a nonlinear transformation. In Example 2.1 (Refer Steven Kay), if we choose to estimate the unknown parameter $\theta = A^2$ by

$$\hat{\theta} = \left(\frac{1}{N}\sum_{n=0}^{N-1} x[n]\right)^2$$

, can we say that the estimator is unbiased? What happens as N -> ∞?

3. The moment generating function is defined as

$$M_x(t) = E(e^{tx}).$$

Given the probability density function

$$f(x) = \begin{cases} e^{-x}, & for\ x > 0 \\ 0 & , elsewhere \end{cases}$$

Compute the moment generating function $M_x(t)$. Expand $M_x(t)$ using McLaurin Series to find μ_2'.

Where μ_2' stands for second order derivative of $M_x''(0)$.