1. For Bernoulli trials, the statistic
$$k = \sum_{n=0}^{N-1} x_n, \quad x_n \in (0,1) \tag{1}$$
is sufficient. Compare the number of bits required to code k with the number of bits required to code the sequence $\mathbf{x} = (x_0, x_1, ..., x_{N-1})$.

2. Let $\mathbf{x} = (x_1, x_2)$ be independent Bernoulli random variables with $P(x_n = 1) = \theta$ and $P(x_n = 0) = (1-\theta)$. Define the order statistic $\mathbf{u} = (u_1, u_2) = (\max(x_1, x_2), \min(x_1, x_2))$. Find $P_\theta(\mathbf{x}|\mathbf{u})$ and show that it is independent of θ.

3. Let $\mathbf{X} = (\mathbf{X}_0, ..., \mathbf{X}_{M-1})$ denote a random sample of random variables $\mathbf{X}_n : N[\mathbf{H}\boldsymbol{\theta}, \mathbf{R}]$. Find the sufficient statistic for $\boldsymbol{\theta}$ and the minimum variance unbiased estimator of $\boldsymbol{\theta}$. What is its variance?

4. In an identification experiment, a known signal sequence $(s_0, s_1, ..., s_t)$ excites a channel with unknown impulse response $(h_0, h_1, ..., h_p)$. The measurement is noisy:
$$\begin{aligned} x_t &= \sum_{i=0}^{p} h_i s_{t-i} + n_t && (2)\\ &= \mathbf{c}_t^T \boldsymbol{\theta} + n_t && (3)\\ \mathbf{c}_t &= [s_t \; ... \; s_{t-p}]^T && (4)\\ \boldsymbol{\theta} &= [h_0 \; ... \; h_p]^T. && (5) \end{aligned}$$
Find a recursion for the sufficient statistic for $\boldsymbol{\theta}$ when the noises n_t are i.i.d. $N[0, \sigma^2]$.

5. Let $\mathbf{X} = (X_0, X_1, ..., X_{M-1})$ denote a random sample of scalar i.i.d. random variables. For each of the following cases, find a sufficient statistic for $\boldsymbol{\theta}$ and find the distribution of the sufficient statistic.
   (a) $X_n : N[\theta_1, 1]$
   (b) $X_n : N[0, \theta_2], \quad \theta_2 > 0$
   (c) $X_n : N[\theta_1, \theta_2], \quad \theta_2 > 0$
   (d) $P[X_n = x] = \binom{N+x-1}{x} (1-p)^N p^x; \quad x = 0, 1, ...$
   (e) $P[X_n = x] = e^{-\lambda} \frac{\lambda^x}{x!}, \quad x = 0, 1, 2, ...$ (Poisson)
   (f) $P[X_n \le x] = \int_0^x [1/\Gamma(\theta_2)\theta_1^{\theta_2}] e^{-y/\theta_1} y^{(\theta_2-1)} dy; \quad 0 \le x < \infty$
   (g) $P[X_n \le x] = \frac{\Gamma(\theta_1+\theta_2)}{\Gamma(\theta_1)\Gamma(\theta_2)} \int_0^x y^{\theta_1-1}(1-y)^{\theta_2-1} dy; \quad 0 \le x \le 1$

6. For each of the cases in the above problem, find the minimum variance unbiased estimate of θ and compute the variance of the estimate.

7. Consider the sample mean and sample variance in a random sample of scalar random variables $X_n : N[m, \sigma^2]$
$$\hat{m} = \frac{1}{M} \sum_{n=0}^{M-1} x_n \tag{6}$$
$$\hat{\sigma}^2 = \frac{1}{M} \sum_{n=0}^{M-1} (x_n - \hat{m})^2. \tag{7}$$

Find the distribution of $\hat{m}$ and of $\hat{\sigma}^2$. Is $\hat{m}$ unbiased? Is $\hat{\sigma}^2$ unbiased? Find minimum variance unbiased estimates of m and σ^2.