EE-602

Assignment 4

[In lieu of Assignment4, tutorials 1,2]

This Assignment will carry a total of 8.75 points

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1. Modify the MATLAB program Monte Carlo in Appendix 2E of Kay's detection theory to determine Pr{ $(1/N)\sum_{n=0}^{N-1} x^2[n] > 5$ } if $x^2[n] \backsim \mathcal{N}(0,5)$, $N = 50$, and the samples are IID. Compare your results against the theoretical probability. Hint: Use the central limit theorem.
2. Assume that we wish to distinguish between the hypotheses $H_0: \boldsymbol{x} \backsim \mathcal{N}(\boldsymbol{0}, \sigma^2\boldsymbol{I})$ and $H_1: \boldsymbol{x} \backsim \mathcal{N}(\boldsymbol{\mu}, \sigma^2\boldsymbol{I})$ based on $\boldsymbol{x} = [x[0]\; x[1]]^{\mathrm{T}}$. If $P(H_0) = P(H_1)$, find the decision regions that minimize P_e. Hint: Show that the decision region boundary is a line that is the perpendicular bisector of the line segment from $\boldsymbol{0}$ to $\boldsymbol{\mu}$.
3. Find the MAP decision rule for

   $$H_0: x[0] \backsim \mathcal{N}(0,1)$$
   $$H_1: x[0] \backsim \mathcal{N}(0,2)$$

   If $P(H_0) = 1/2$ and also if $P(H_0) = 3/4$. Display the decision regions in each case and explain.
4. A detection probability P_D is to be determined via a Monte Carlo computer simulation. It is known that $P_D \geq 0.8$. If the relative absolute error is to be no more than 0.01 for 95% of the time, how many Monte Carlo trials are required?
5. We wish to detect a DC level A in WGN with variance σ^2 based on the samples $x[n]$ for $n = 0,1,2 \ldots, N-1$. The amplitude of the DC level A is known but the variance σ^2 of the noise is unknown. Use a Bayesian approach with the prior PDF

   $$p(\sigma^2) = \begin{cases} \frac{\lambda \exp(-\lambda/\sigma^2)}{\sigma^4}, & \sigma^2 > 0 \\ 0, & \sigma^2 < 0 \end{cases}$$

   where $\lambda > 0$ .Find the detector that maximizes P_D for a fixed P_{FA}. Do not evaluate the threshold. Explain what happens as $\lambda \to 0$. Hint: You will need the Gamma integral

   $$\int_0^\infty x^{b-1} \exp(-ax)\, dx = a^{-b}\Gamma(b)$$
6. Find the NP test to distinguish between the hypotheses that a single sample $x[0]$ is observed from the possible PDFs

   $$H_0: p(x[0]) = \frac{1}{2} exp\,(-|x[0]|)$$
   $$H_1: p(x[0]) = \frac{1}{\sqrt{2\pi}} exp\,(-x^2[0])$$

   Show the decision regions. Hint: You will need to solve a quadratic inequality.