Assignment No. 3

1. If x[n] for $n = 0,1,\ldots,N-1$ are IID according to $U[0,\theta]$, show that the regularity condition does not hold or that

$$E\left[\frac{\partial \ln p(x;\ \theta)}{\partial\theta}\right] \neq 0 \qquad for\ all\ \theta > 0.$$

Hence, the CRLB cannot be applied to this problem.

2. If $x[n] = A + w[n]$ for $n = 0,1,\ldots,N-1$ are observed and $\boldsymbol{w} = [w[0]\ w[1] \ldots .\ w[N-1]]^T \sim N(\mathbf{0}, \boldsymbol{C})$, find the CRLB for A. Does an efficient estimator exist and if so what is its variance?

3. We observe two samples of a DC level in *correlated* Gaussian noise

$$\begin{aligned} \mathrm{x}[0] &= A + \mathrm{w}[0] \\ \mathrm{x}[1] &= A + \mathrm{w}[1] \end{aligned}$$

where $\boldsymbol{w} = [w[0]\ w[1]]^T$ is zero mean with covariance matrix

$$C = \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}.$$

The parameter ρ is the correlation coefficient between w[0] and w[1]. Compute the CRLB for A and compare it to the case when w[n] is WGN or $\rho = 0$. Also, explain what happens when $\rho \rightarrow \pm 1$. Finally comment on the additivity property of the Fisher information for non-independent observations.

4. For a 2×2 Fisher information matrix

$$\boldsymbol{I}(\theta) = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

which is positive definite, show that

$$[\boldsymbol{I}^{-1}(\theta)]_{11} = \frac{c}{ac - b^2} \geq \frac{1}{a} = \frac{1}{[\boldsymbol{I}(\theta)]_{11}}$$

What does this say about estimating a parameter when a second parameter is either known or unknown? When does equality hold and why?