Assignment No. 2

1. Using a moment generating function, show that the linear transformation of a Gaussian random vector is also a Gaussian.

2. Let $\{x_k(\zeta)\}_{k=1}^{4}$ be four IID random variables with exponential distribution (P.1, Manolakis, Chapter 3) with a = 1. Let

$$y_k(\zeta) = \sum_{l=1}^{k} x_k(\zeta) \qquad 1 \leq k \leq 4$$

(a) Determine and plot the pdf of $y_2(\zeta)$.
(b) Determine and plot the pdf of $y_3(\zeta)$.
(c) Determine and plot the pdf of $y_4(\zeta)$.
(d) Compare the pdf of $y_4(\zeta)$ with that of the Gaussian density.

3. A causal LTI system, which is described by the difference equation

$$y(n) = \frac{1}{2}y(n-1) + x(n) + \frac{1}{3}x(n-1)$$

is driven by a zero-mean WSS process with autocorrelation $r_x(l) = 0.5^{|l|}$.

(a) Determine the PSD and autocorrelation of the output sequence $y(n)$.
(b) Determine the cross-calibration $r_{xy}(l)$ and cross-PSD $R_{xy}(e^{j\omega})$ between the input and the output signals.

(Illustrate Results in MATLAB)

4. If X has the probability density

$$f(x) = \begin{cases} k.\,e^{-3x} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

find k and $P(0.5 \leq X \leq 1)$.

5. Find a probability density function for a random variable whose distribution function is given by

$$F(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ x & \text{for } 0 < x < 1 \\ 1 & \text{for } x \geq 1 \end{cases}$$

And plot its graph.

6. Find the joint probability density of the two random variables X and Y whose joint distribution function is given by

$$F(x, y) = \begin{cases} (1 - e^{-x})(1 - e^{-y}) & \text{for } x > 0 \text{ and } y > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Also use the joint probability density to determine $P(1 < X < 3, 1 < Y < 2)$.

7. Given the independent random variables X_1, X_2 and X_3 with the probability densities

$$f_1(x_1) = \begin{cases} e^{-x_1} & \text{for } x_1 > 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$f_1(x_1) = \begin{cases} 2e^{-2x_2} & \text{for } x_2 > 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$f_1(x_1) = \begin{cases} 3e^{-3x_3} & \text{for } x_3 > 0 \\ 0 & \text{elsewhere} \end{cases}$$

find their joint probability density, and use it to evaluate the probability $P(X_1 + X_2 \leq 1, \; X_3 > 1)$.

8. Show that

$$E[(aX + b)^n] = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i E(X^{n-i})$$

9. Find the moment generating function of the random variable whose probability density is given by

$$f(x) = \begin{cases} e^{-x} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

And use it to find an expression for μ_r'.